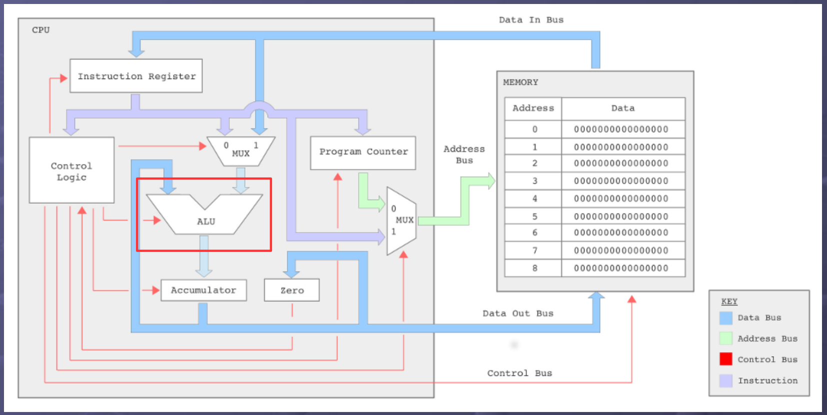
Systems and Devices 1
Lec 3b :
Combinatorial Logic

SimpleCPU_v1a

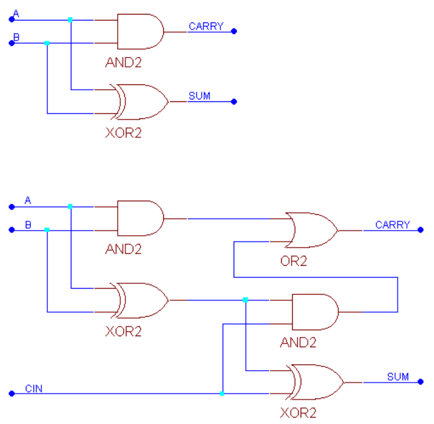


- Block diagram
  - ALU : a core requirement of any computer is to process data i.e. the Arithmetic and Logic unit, the ADDER the heart of any CPU.

Binary addition

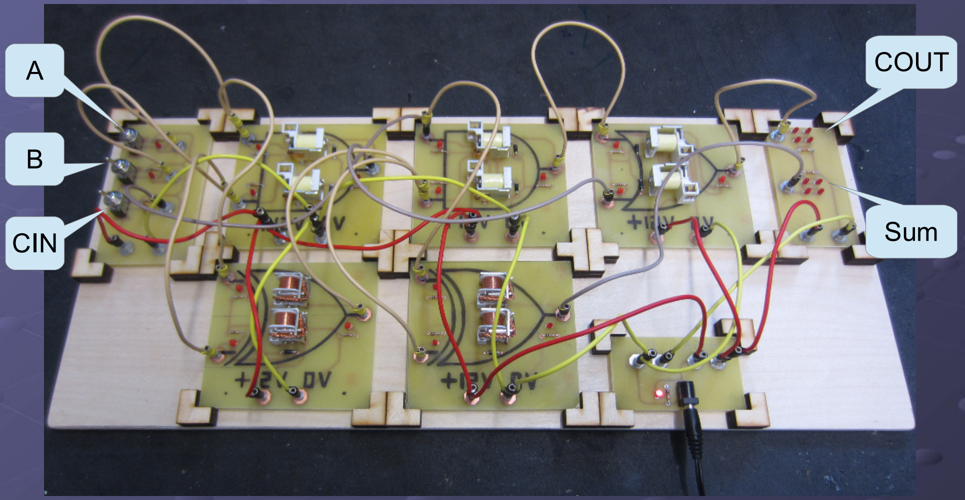
A	B	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

A	B	Cin	Carry	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



- Half and full adder
  - Basic components can be combined into larger circuits

Demo : relay logic



- Full adder

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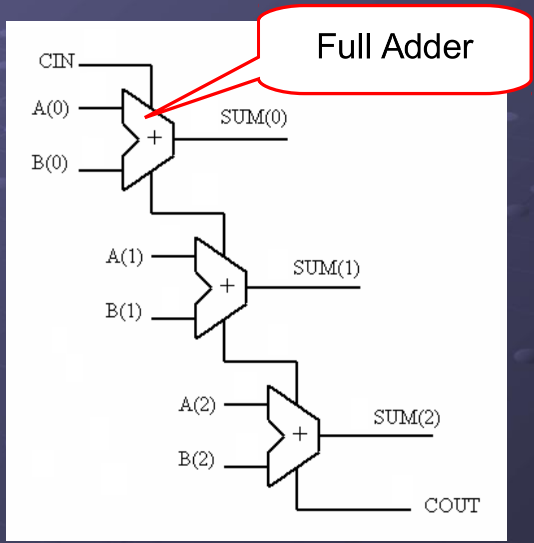
Binary addition

- Ripple adder
  - Replicated full adders
    - Three FA, producing a 3 bit adder
  - LSB carry in (CIN) is set to zero
  - Carry out (COUT) feeds carry signal to next full adder stage

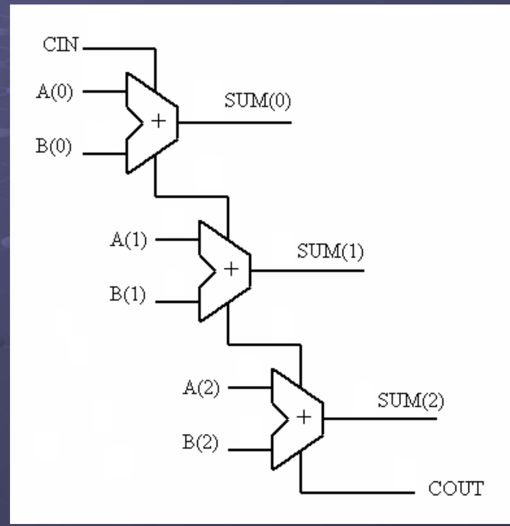


Slide 6

Binary addition

- Ripple adder
  - Add : 7 + 1

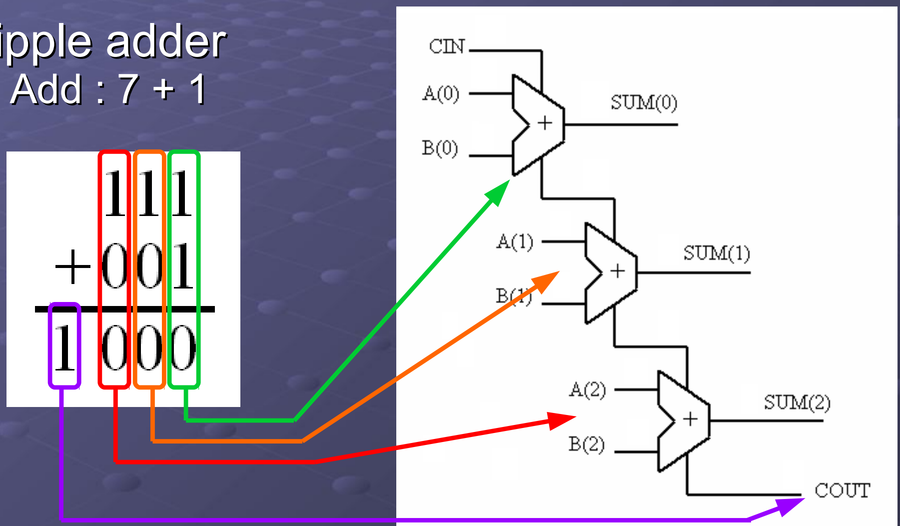
$$\begin{array}{r} 111 \\ +001 \\ \hline 1\,000 \end{array}$$



Slide 7

Binary addition

- Ripple adder
  - Add : 7 + 1



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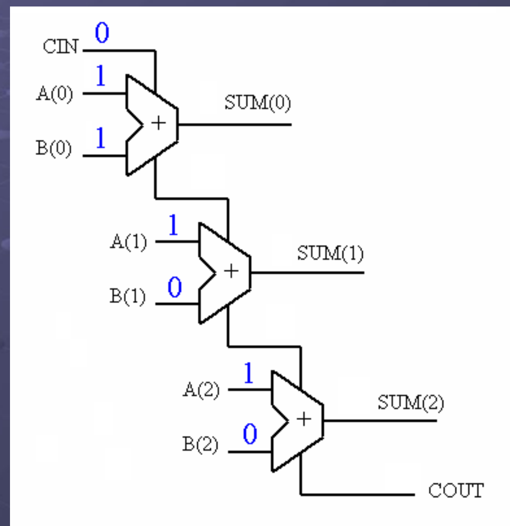
Binary addition

- Ripple adder
  - Add : 7 + 1

$$\begin{array}{r} 111 \\ +001 \\ \hline 1\,000 \end{array}$$



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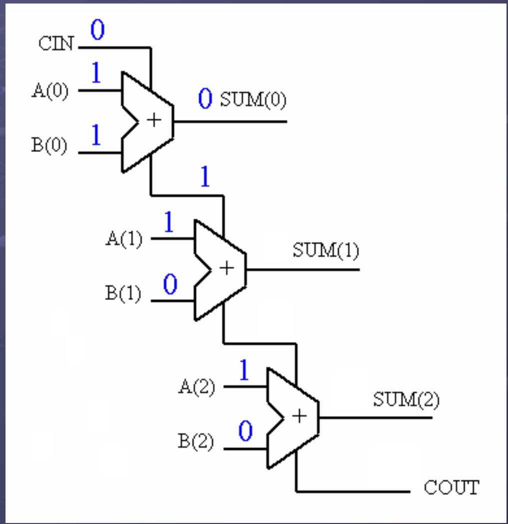
Binary addition

- Ripple adder
  - Add : 7 + 1

111
+001
1 000



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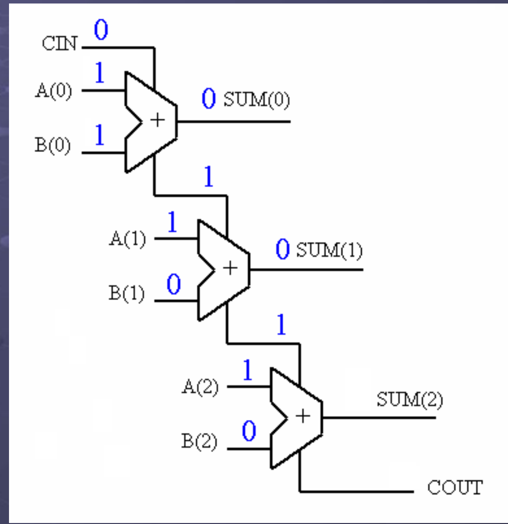
Binary addition

- Ripple adder
  - Add : 7 + 1

111
+001
1 000



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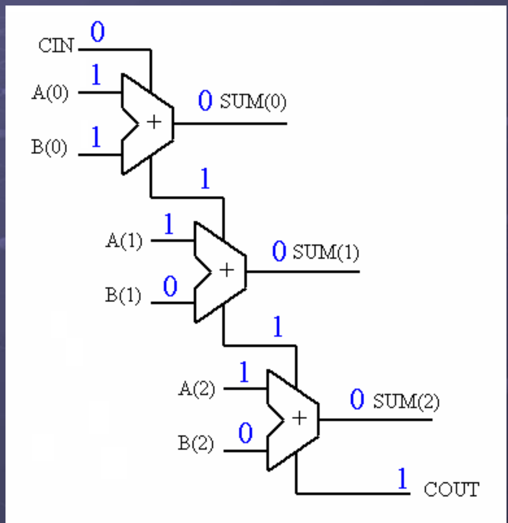
Binary addition

- Ripple adder
  - Add : 7 + 1

111
+001
1 000

  - Result 8_{10}, 1000_2



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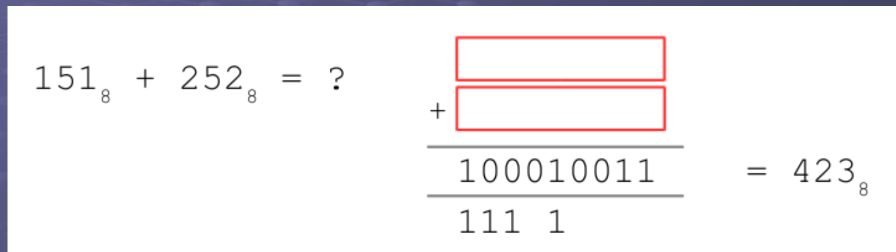
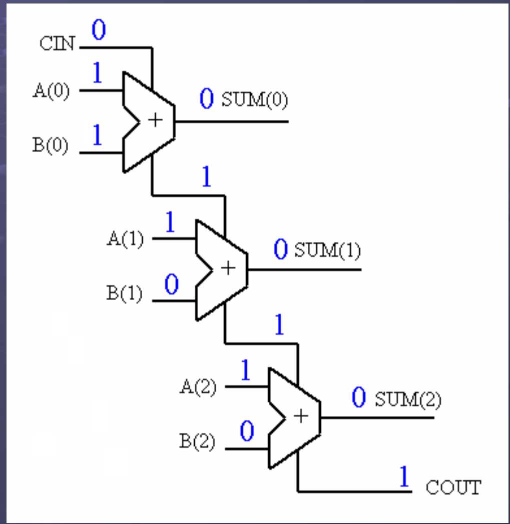
Binary addition



- Quick Quizzz
  - Convert the following values into binary then confirm the result of the binary addition.
  - Is the conversion of the binary result to octal correct?

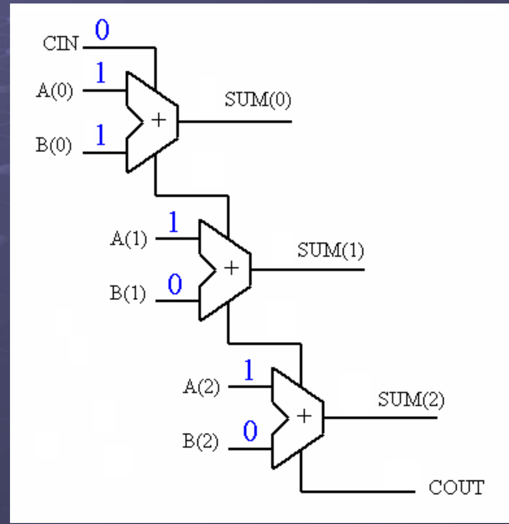
Binary addition

- Ripple adder
  - MSB Carry Out
    - Can be passed to additional full adder stage to allow larger adders to be constructed.
    - Can be used to indicate that the result has exceeded the maximum bit representation i.e. an ***overflow*** has occurred.
  - **Important**, will use these ideas when writing assembly code.



Binary addition

- Ripple adder
  - Remember that hardware is not software i.e. each full adder will operate in parallel.
  - The result will go through a series of states before it settles down to the final value.

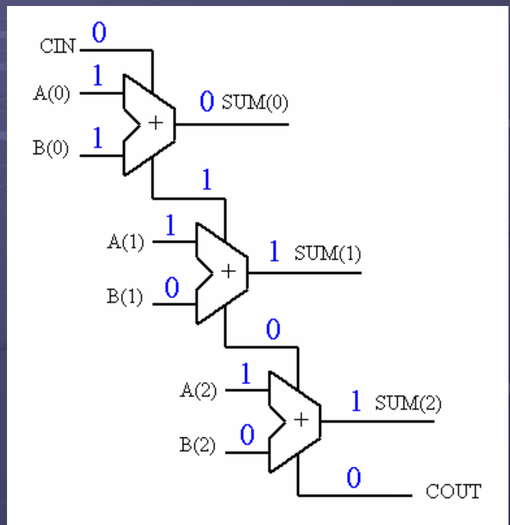


Binary addition

- Ripple adder
  - Add : 7 + 1
  - Step 1

```
  111
+001
0 110
```

  - Result 6_{10},0110_2

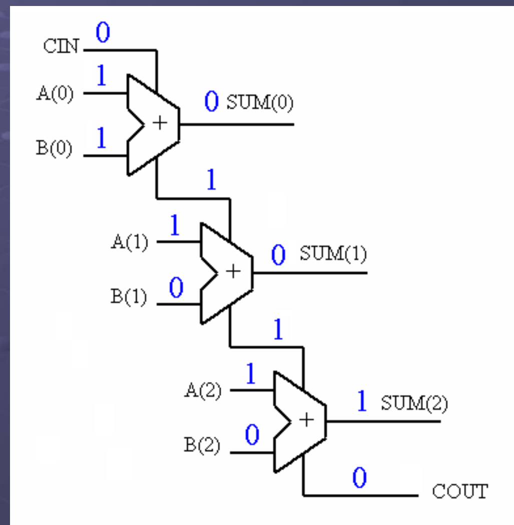


Binary addition

- Ripple adder
  - Add : 7 + 1
  - Step 2

```
  111
+001
0 100
```

  - Result 4_{10},0100_2



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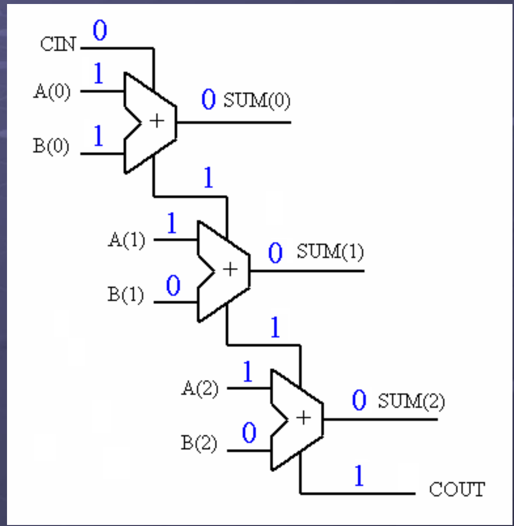
Binary addition

- Ripple adder
  - Add : 7 + 1
  - Step 3

$$\begin{array}{r} 111 \\ +001 \\ \hline 1\,000 \end{array}$$

  - Result 8_{10}, 1000_2



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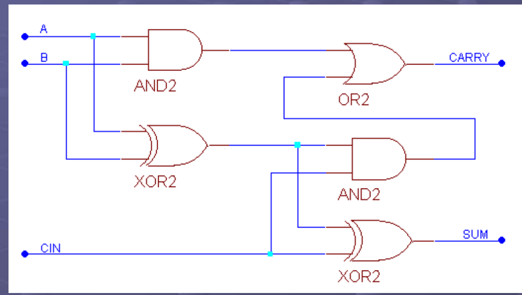
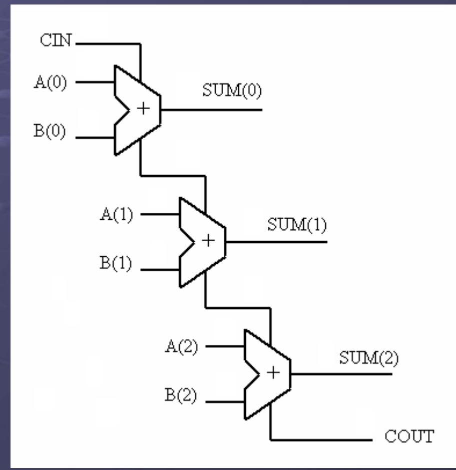
Binary addition





- Quick Quizz
  - If each logic gate takes 10 ns to process a signal, what is the critical path delay of this ripple adder?

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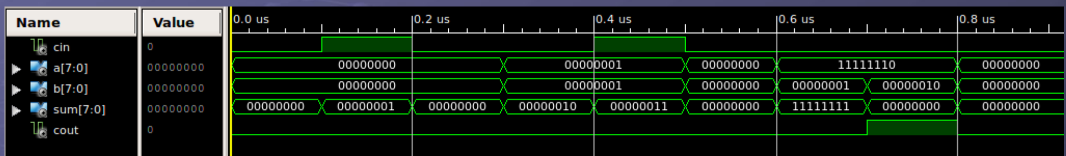
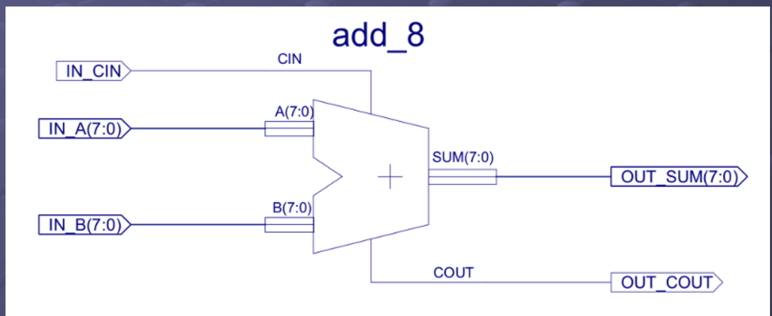
Example : adder_8.zip





- 8 bit ripple adder

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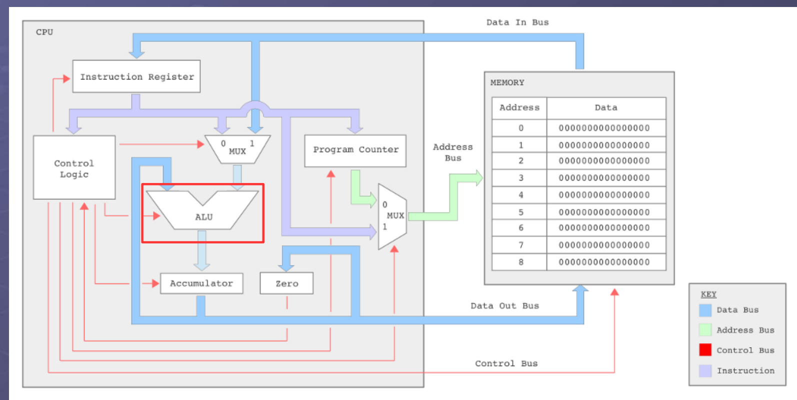
SimpleCPU_v1a



- Block diagram
  - Q : how do we control the ALU's function e.g. pass through, add, subtract and bitwise AND functions, as defined in the instruction set? How do we implement these functions?

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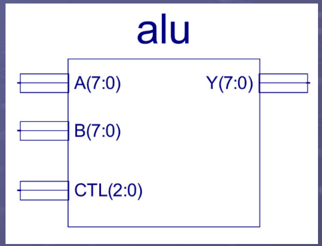
ALU



ALU CTL2	ALU CTL1	ALU CTL0	OP
0	0	0	ADD
0	0	1	SUB
0	1	0	AND
0	1	1	NU
1	0	0	PASS
1	0	1	NU
1	1	0	NU
1	1	1	NU

- ALU interface and control (CTL) signals
  - A(7:0) – 8bit input, driven by ACC
  - B(7:0) – 8bit input, driven by Data MUX, IR(7:0) or DIN(7:0)
  - CTL(2:0) – 3bit input, function select, driven by control logic
  - Y(7:0) – 8bit output, result of selected function, Y <= A op B.
- Pass through = multiplexer, addition = ripple adder
- How do we perform subtraction and bitwise AND?

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Key skills : working in base 2

```
 10010110     150
-00101100     -44
_________     ___

_________     ___
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

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Key skills : working in base 2

```
 10010110     150
-00101100     -44
_________     ___
        0
_________     ___
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

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Key skills : working in base 2

```
 10010110     150
-00101100     -44
_________     ___
       10
_________     ___
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

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Key skills : working in base 2

10010110	150
-00101100	-44
010	

- Subtract two binary numbers : 150 - 44
  - Positive, integer

Slide 26

Key skills : working in base 2

10010110	150
-00101100	-44
010	

- Subtract two binary numbers : 150 - 44
  - Positive, integer

Slide 27

Key skills : working in base 2

- Borrow case
  - Look to the left until the first 1 is found, this defining a block i.e. 10…0
  - Write a 1 in the result and update block to 01….1
  - Continue subtraction
- Alternatively, another way to think of it
  - Borrow '2' from the left column
    - Same process you would perform in base 10, but rather than borrowing 10 you borrow 2

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Key skills : working in base 2

10010110	150
-00101100	-44
010	

- Subtract two binary numbers : 150 - 44
  - Positive, integer

Slide 29

Key skills : working in base 2

```
  10001110        150
 -00101100        -44
 ---------        ---
      1010
 ---------        ---
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

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Key skills : working in base 2

```
  10001110        150
 -00101100        -44
 ---------        ---
     01010
 ---------        ---
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

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Key skills : working in base 2

```
  10010110        150
 -00101100        -44
 ---------        ---
     01010
 ---------        ---
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

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Key skills : working in base 2

```
  01110110        150
 -00101100        -44
 ---------        ---
    101010
 ---------        ---
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

Key skills : working in base 2

```
 01110110     150
-00101100     -44
---------    ----
  1101010
---------    ----
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

Key skills : working in base 2

```
 01110110     150
-00101100     -44
---------    ----
 01101010
---------    ----
```

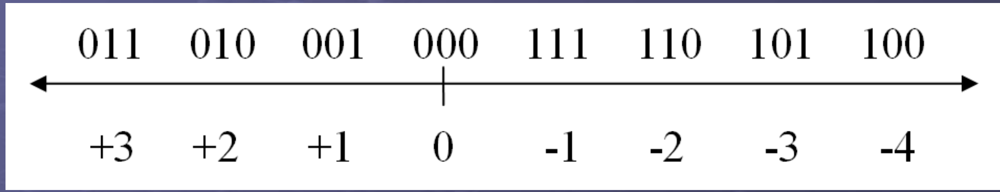
- Subtract two binary numbers : 150 - 44
  - Positive, integer

Key skills : working in base 2

```
 10010110     150
-00101100     -44
---------    ----
 01101010     106
---------    ----
```

- Subtract two binary numbers : 150 - 44
  - Positive, integer

Method of Complements



- Q : How do we represent negative numbers?
  - Using the complement of a number e.g. 2s complement
  - MSB represents the sign: 0 = +num, 1 = –num
    - Max positive sign bit = 0, MSB –1 to LSB = 1
    - Max negative sign bit = 1, MSB –1 to LSB = 0
- To convert to a Two's complement representation
  - Invert each bit position (one's complement) 0→1, 1→0
  - Add 1 (carry ignored)

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2s Complement

$1_{10} = 00000001_2$

One's Complement : 11111110
Add one : 11111111

$-1_{10} = 11111111_2$

$100_{10} = 01100100_2$

One's Complement : 10011011
Add one : 10011100

$-100_{10} = 10011100_2$

$200_{10} = 11001000_2$

One's Complement : 00110111
Add one : 00111000

$-200_{10} = 00111000_2$???

- Examples
  - MSB represents the number's sign i.e. a signed number.
  - Maximum value that can be represented is halved compared to an unsigned representation

Slide 38

2s Complement

$-100_{10} = 10011100_2$

One's Complement : 01100011
Add one : 01100100

$100_{10} = 01100100_2$

- To determine the absolute value of a negative binary number
  - Take the Two's complement again

Eight bit signed number	Sixteen bit signed number
$-100_{10} = 10011100_2$	$-100_{10} = 1111111110011100_2$
$100_{10} = 01100100_2$	$100_{10} = 0000000001100100_2$

- Note, when changing the size of a number don't forgot to sign extend.

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Binary subtraction

0x2A − 0x6C = ?

[ ]
− [ ]
10111110 = 0xBE

0x2A
− 0x6C
0xBE

42_{10} − [ ] =

10111110
01000001
01000010 = -66_{10}

- Quick Quizzz
  - Convert the following values into binary then confirm the result of the binary subtraction.
  - Is the conversion of the binary result to hexadecimal correct?

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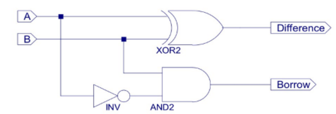
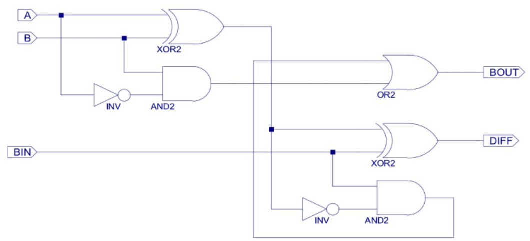
Binary subtraction

A	B	BORROW	DIFFERENCE
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0



A	B	BIN	BOUT	DIFF
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



- We could implement the subtraction operation using half and full subtractors, but …

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Key skills : working in base 2

010010110	150
-000101100	-44

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

010010110	150
+111010100	-44

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

010010110	150
+111010100	-44
0	

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

010010110	150
+111010100	-44
10	

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
 010010110     150
+111010100     -44
----------    ----
       010
----------    ----
      1
```

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
 010010110     150
+111010100     -44
----------    ----
      1010
----------    ----
      1
```

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
 010010110     150
+111010100     -44
----------    ----
     01010
----------    ----
    1 1
```

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
 010010110     150
+111010100     -44
----------    ----
    101010
----------    ----
    1 1
```

- Using the Two's complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
  010010110      150
 +111010100      -44
 ----------      ---
    1101010
 ----------      ---
     1  1
```

- Using the Two’s complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
  010010110      150
 +111010100      -44
 ----------      ---
   01101010
 ----------      ---
  1   1  1
```

- Using the Two’s complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
  010010110      150
 +111010100      -44
 ----------      ---
  001101010      106
 ----------      ---
 11   1  1
```

- Using the Two’s complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Key skills : working in base 2

```
  010010110      150
 +111010100      -44
 ----------      ---
  001101010      106
 ----------      ---
 11   1  1
```

- Using the Two’s complement representation simplifies binary subtraction i.e. can be performed using addition
  - A - B = A + (-B)

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Addition of negative numbers

- When using Two's complement representation the carry bit can no longer be used to indicate an overflow.
  - Oveflow - number (result) can not be represented by the maximum number of bits within a memory location or register i.e. need more bits, can not be stored.
- Overflow is determined by these rules
  - If operand sign bits are equal then result sign bit must equal operand sign bit
    - E.g. (A + B) or (-A + -B) magnitude always bigger
  - If operand sign bit are not equal then overflow can not occur
    - E.g. (A – B) or (-A + B) magnitude always smaller

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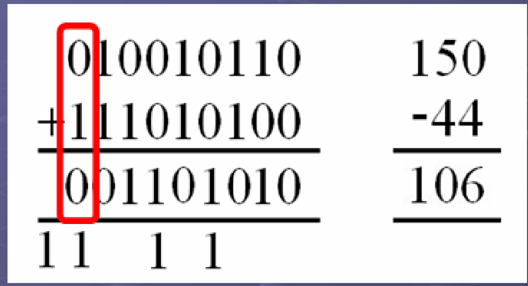
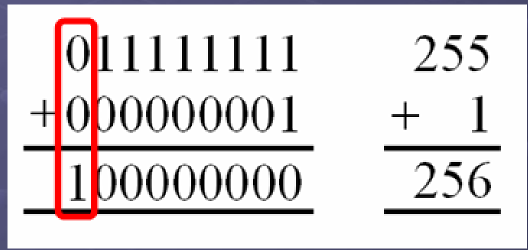
Addition of negative numbers



- Different sign bits
  - Overflow can not occur



- Matching sign bits
  - Overflow may occur

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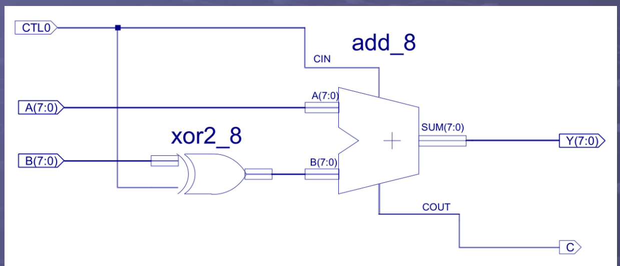
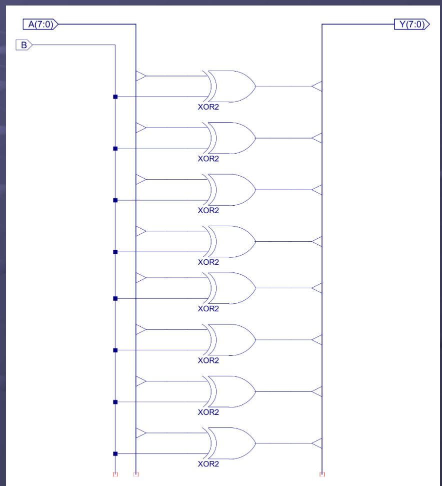
Adder / Subtractor unit





- How do we perform 2's complement in hardware?
  - ADD_SUB_8
    - Ripple adder
    - XOR array

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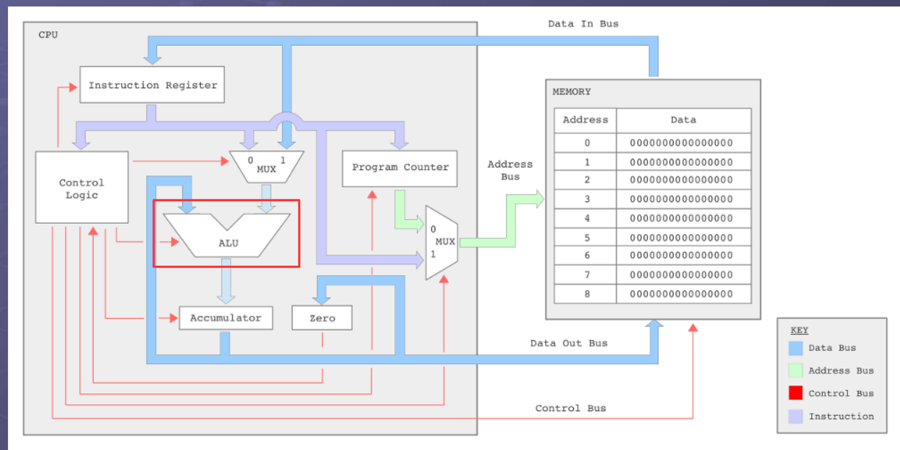
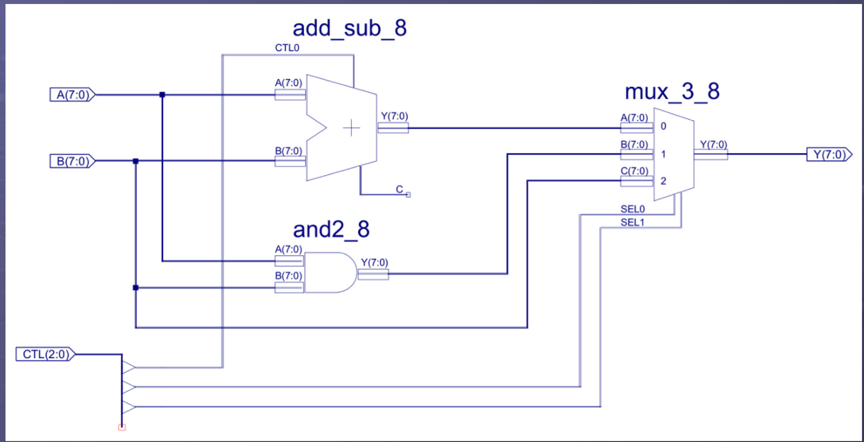
SimpleCPU_v1a



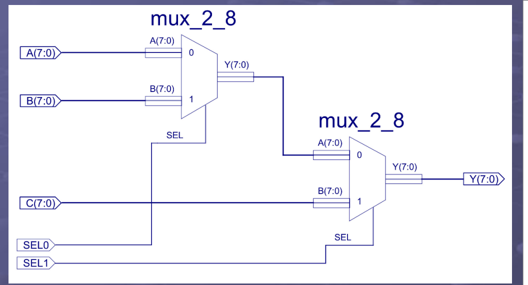
- Block diagram
  - Q : what else is inside the ALU?

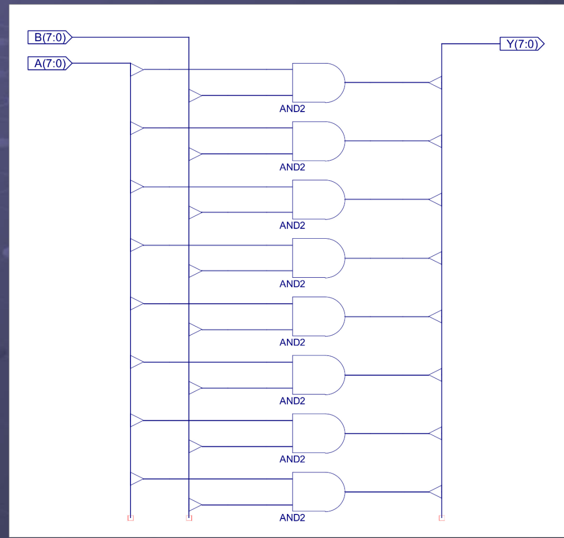
ALU



- A : not a lot, a simple ALU for a simple instruction set
  - Q : what will happen to the CPD if we have more, "complex" (multiply, divide, square root) instructions?

ALU

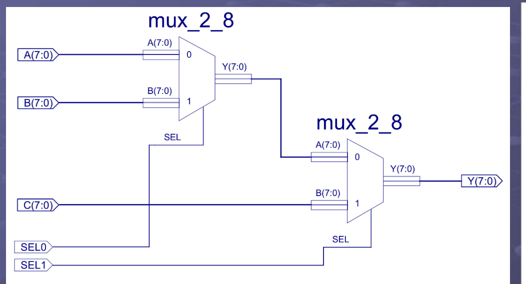


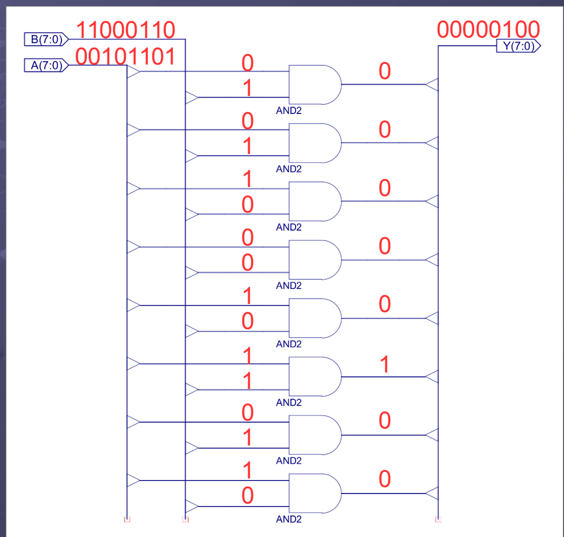


- Bitwise AND

A	00101101	45
B	11000110	198 or -58
	00000100	4

ALU





- Bitwise AND

A	00101101	45
B	11000110	198 or -58
	00000100	4

Summary

- Key concepts :
  - Binary arithmetic
    - Half subtractor, Full subtractor and Ripple subtractor
    - Representing negative numbers - 2's complement
    - Subtraction using addition of a negative number
      - Detecting overflows when using signed data
  - Data processing : Arithmetic & Logic Unit (ALU)
    - Pass, Add, Subtract and Bitwise AND
    - More complex functions can be construction in software using these basic binary operators e.g. multiply, divide etc.